TEACHING THE METHOD OF MATHEMATICAL INDUCTION

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Abstract: This scientific article focuses on the method of mathematical induction for in-depth group students of academic lyceums, independent learners.

Keywords: Mathematical induction principle, mathematical induction base, mathematical induction hypothesis, mathematical induction step.

O`zbekiston respublikasida ta`lim – davlat siyosatining asosiy yo`nalishlaridan biri sifatida qaralmoqda. Chunki, global o`zgarishlar davrida ta`lim-tarbiyaga ehtiyoj ortib, hayot o`zgarish sur`atlari tezlashmoqda.

Ushbu maqola mustaqil o`rganuvchi shaxs dunyoqarashini kengaytirishda, ijodkor, ijtimoiy faol, kreativ, ma`naviy boy shaxsni shakllantiruvchi va yuqori malakali raqobotdosh kadrlar tayyorlashda yordam beradi degan umiddamiz.

Matematik induksiya usuli.

Matematik induksiya usuli asosida matematik induksiya prinsipi yotadi. Bu prinsipning mazmunini izohlaylik. Biror n natural songa bog`liq bo`lgan fikrimizni $A(n)$ orqali belgilaylik. Bu fikrimizning ixtiyoriy n natural son uchun to`g`riligini barcha n uchun bevosita tekshirib ko`rishning iloji bo`lmasin. $A(n)$ mulohaza matematik induksiya prinsipiga ko`ra quyidagicha isbotlanadi:

1) $A(n)$ mulohazaning to`g`riligi, avvalo, $n = 1$ uchun tekshiriladi.
2) Aytilgan mulohaza $n = k$ uchun to`g`ri deb faraz qilinadi
3) Qilingan farazdan foydalanib, mulohazaning $n = k + 1$ uchun to`g`riligi isbotlanadi.

Shundan so`ng, $A(n)$ mulohaza barcha $n \in N$ uchun isbotlangan hisoblanadi. Shunday qilib, matematik induksiya prinsipiga asoslangan isbot **matematik induksiya usuli** bilan isbotlash deyiladi.

Matematik induksiya usuli yordamida misollar yechish.

1-misol. *$a \in N$ bo`lsa,* $(a^5 - 5a^3 + 4a) \vdots 120$ bo`lishni isbotlang.

Isboti: 1) $a = 1$ uchun tasdiq to`g`ri, ya'ni $(1^5 - 5 \cdot 1^3 + 4 \cdot 1) \vdots 120$.

2) Faraz qilaylik, $a = k$ bo`lganda $(k^5 - 5k^3 + 4k) \vdots 120$ bo`lsin.

3) $a = k + 1$ bo`lganda $[(k+1)^5 - 5(k+1)^3 + 4(k+1)] \vdots 120$ bo`lishini isbotlaymiz:

$$[(k+1)^5 - 5(k+1)^3 + 4(k+1) = (k+1) \cdot [(k+1)^4 - 5(k+1)^2 + 4] =$$
$$= (k+1) \cdot [(k+1)^2 - 4] \cdot [(k+1)^2 - 1] = (k+1) \cdot (k^2 + 2k) \cdot (k^2 + k - 3) =$$

$= (k^3 + 3k^2 + 2k) \cdot (k^2 + 2k - 3) = (k^5 - 5k^3 + 4k) + 5(k^4 + 2k^3 - k^2 - 2k)$ Bunda farazga ko`ra $(k^5 - 5k^3 + 4k)\vdots 120$.

Endi $(k^4 + 2k^3 - k^2 - 2k)\vdots 24$ ekanligini ko`rsata olsak, tasdiqning to`g`riligini isbotlagan bo`lamiz: $k^4 + 2k^3 - k^2 - 2k = k \cdot [k^2(k+2) - (k+2)] = k \cdot (k+2) \cdot (k^2 - 1) = \quad = (k-1) \cdot k \cdot (k+1) \cdot (k+2)$ Bu oxirgi ko`paytma ketma-ket keluvchi 4 ta natural sonning ko`paytmasi bo`lib, tarkibida hamma vaqt $24$ ko`paytuvchi bo`ladi. Demak, $k \in N$ bo`lganda $(k^4 + 2k^3 - k^2 - 2k)\vdots 24$ bo`ladi. Berilgan tasdiq istalgan $n$ natural son uchun o`rinli ekan. **Isbot tugadi.**

2-misol. $n \in N$ bo`lsa, $1 + 2 + 3 + \cdots + n = \frac{n\cdot(n+1)}{2}$ tenglikning tog`riligini isbotlang.

Isboti: 1) $n = 1$ uchun tasdiq to`g`ri, ya'ni $1 = \frac{1\cdot(1+1)}{2} \Rightarrow 1 = 1$

2) Faraz qilaylik, $n = k$ bo`lganda $1 + 2 + 3 + \cdots + k = \frac{k\cdot(k+1)}{2}$ tenglik to`g`ri bo`lsin.

3) $n = k + 1$ bo`lganda $1 + 2 + 3 + \cdots + k + (k+1) = \frac{(k+1)\cdot(k+2)}{2}$ tenglikning to`g`riligini isbotlaymiz:

$$1 + 2 + 3 + \cdots + k + (k+1) = (1 + 2 + 3 + \cdots + k) + (k+1) =$$
$$= \frac{k\cdot(k+1)}{2} + (k+1) = \frac{k\cdot(k+1) + 2(k+1)}{2} = \frac{(k+1)\cdot(k+2)}{2}.$$ Isbot tugadi.

3-misol. n ning barcha natural qiymatlarida

$1^3 + 2^3 + 3^3 + \cdots + n^3 = (1 + 2 + 3 + \cdots + n)^2$ tenglikning to`g`ri ekanligini isbotlang.

Isboti: Matematik induksiya usulidan foydalanamiz:

1) $n = 1$ uchun tasdiq to`g`ri, ya'ni $1^3 = (1)^2$.

2) $n = k$ bo`lganda $1^3 + 2^3 + 3^3 + \cdots + k^3 = (1 + 2 + 3 + \cdots + k)^2$ tenglik to`g`ri deb faraz qilamiz.

3) Farazdan foydalanib, $n = k + 1$ bo`lganda

$1^3 + 2^3 + 3^3 + \cdots + (k+1)^3 = (1 + 2 + 3 + \cdots + (k+1))^2$ tenglikning tog`riligini isbotlaymiz:

$$1^3 + 2^3 + 3^3 + \cdots + (k+1)^3 = (1^3 + 2^3 + 3^3 + \cdots + k^3) + (k+1)^3 =$$
$$=(1 + 2 + 3 + \cdots + k)^2 + (k+1)^3 = \left(\frac{k\cdot(k+1)}{2}\right)^2 + (k+1)^3 = \frac{k^2\cdot(k+1)^2}{4} + (k+1)^3$$

Bundan $(k+1)^2$ ni qavsdan chiqaramiz, u holda $(k+1)^2 \cdot \left(\frac{k^2}{4} + (k+1)\right) =$

$$=(k+1)^2 \cdot \left(\frac{k^2}{4} + 2 \cdot \frac{k}{2} + 1\right) = (k+1)^2 \cdot \left(\frac{k}{2} + 1\right)^2 = \left((k+1) \cdot \left(\frac{k}{2} + 1\right)\right)^2 =$$

$= \left(\frac{(k+1)(k+2)}{2}\right)^2 =$ bu esa $(1 + 2 + 3 + \cdots + (k+1))^2$ ga teng.

4-misol. $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n \cdot (n+1) = \frac{n\cdot(n+1)\cdot(n+2)}{3}$ tenglik n ning barcha natural qiymatlarida to`g`ri bo`lishini isbotlang.

Isboti: Matematik induksiya usulidan foydalanib isbotlaymiz:

1) $n = 1$ uchun tenglik to`g`ri bo`ladi, ya'ni $1 \cdot 2 = \frac{1\cdot(1+1)\cdot(1+2)}{3} = \frac{1\cdot2\cdot3}{3} \Rightarrow 2=2$

2) $n = k$ bo`lganda $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + k \cdot (k+1) = \frac{k\cdot(k+1)\cdot(k+2)}{3}$ tenglik to`g`ri deb faraz qilamiz.

3) Farazdan foydalanib, $n = k + 1$ bo`lganda

$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (k+1) \cdot (k+2) = \frac{(k+1)\cdot(k+2)\cdot(k+3)}{3}$ tenglikning tog`riligini isbotlaymiz:

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (k+1) \cdot (k+2) =$$

$= [1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + k \cdot (k+1)] + (k+1) \cdot (k+2) =$ farazdan foydalansak, $= \frac{k\cdot(k+1)\cdot(k+2)}{3} + (k+1) \cdot (k+2) =$ bundan $(k+1) \cdot (k+2)$ ni qavsdan chiqaramiz, natijada $= (k+1) \cdot (k+2) \cdot \left(\frac{k}{3} + 1\right) = \frac{(k+1)\cdot(k+2)\cdot(k+3)}{3}$ tenglikning to`g`riligini isbotladik. Isbot tugadi.

5-misol. $1 + 3 + 5 + 7 + \cdots + (2n-1) = n^2$ tenglik n ning barcha natural qiymatlarida to`g`ri ekanligini isbotlang.

Isboti: Matematik induksiya usulidan foydalanib isbotlaymiz:

1) $n = 1$ uchun tenglik to`g`ri bo`ladi, ya'ni $1 = 1^2$

2) $n = k$ bo`lganda $1 + 3 + 5 + 7 + \cdots + (2k-1) = k^2$ tenglik to`g`ri deb faraz qilamiz.

3) Farazdan foydalanib, $n = k + 1$ bo`lganda

$1 + 3 + 5 + 7 + \cdots + (2k+1) = (k+1)^2$ tenglikning tog`riligini isbotlaymiz:

$1 + 3 + 5 + 7 + \cdots + (2k+1) = [1 + 3 + 5 + 7 + \cdots + (2k-1)] + (2k+1) = k^2 + (2k+1) = (k+1)^2$ Isbot tugadi.

6-misol. $\frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \frac{1}{7\cdot9} + \cdots + \frac{1}{(2n+1)\cdot(2n+3)} = \frac{n}{3\cdot(2n+3)}$ tenglik n ning barcha natural qiymatlarida to`g`ri ekanligini isbotlang.

Isboti: Bu tenglikni ham matematik induksiya usuli orqali isbotlash qulay:

1) $n = 1$ uchun tenglik to`g`ri bo`ladi, ya'ni $\frac{1}{3\cdot5} = \frac{1}{3\cdot(2\cdot1+3)} \Rightarrow \frac{1}{3\cdot5} = \frac{1}{3\cdot(2+3)} \Rightarrow \frac{1}{15} = \frac{1}{15}$

2) $n = k$ bo`lganda $\frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \frac{1}{7\cdot9} + \cdots + \frac{1}{(2k+1)\cdot(2k+3)} = \frac{k}{3\cdot(2k+3)}$ tenglik to`g`ri deb faraz qilamiz.

3) Farazdan foydalanib, $n = k + 1$ bo`lganda

$\frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \frac{1}{7\cdot9} + \cdots + \frac{1}{(2k+3)\cdot(2k+5)} = \frac{k+1}{3\cdot(2k+5)}$ tenglikning to`g`riligini isbotlaymiz: $\frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \frac{1}{7\cdot9} + \cdots + \frac{1}{(2k+3)\cdot(2k+5)} = \left(\frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \frac{1}{7\cdot9} + \cdots + \frac{1}{(2k+1)\cdot(2k+3)}\right) + \frac{1}{(2k+3)\cdot(2k+5)} == \frac{k}{3\cdot(2k+3)} + \frac{1}{(2k+3)\cdot(2k+5)} = \frac{k\cdot(2k+5)+3}{3\cdot(2k+3)\cdot(2k+5)} =$

$= \frac{1}{2k+3} \cdot \frac{2k^2+5k+3}{3\cdot(2k+5)} = \frac{1}{2k+3} \cdot \frac{2k^2+2k+3k+3}{3\cdot(2k+5)} = \frac{1}{2k+3} \cdot \frac{2k(k+1)+3(k+1)}{3\cdot(2k+5)} =$

$= \frac{1}{2k+3} \cdot \frac{(2k+3)\cdot(k+1)}{3\cdot(2k+5)} = \frac{k+1}{3\cdot(2k+5)}$ Isbot tugadi.

O`zingizni sinab ko`ring.

1. Ketma ket keluvchi 3 ta natural son kublarining yig'indisi 9 ga bo'linishini isbotlang.
2. Agar $n \in N$ bo'lsa , $(3^{2n+1} + 40n - 67) \vdots 64$ bo'lishini isbotlang.

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