SCHOLASTICO-2021

ON THE TIME OPTIMAL CONTROL PROBLEM FOR CONTROLLABLE DIFFERENTIAL INCLUSION WITH PARAMETER

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Abstract:

In this paper the controllable differential inclusions with parameters is considered. The time optimal control problem for ensemble of trajectories is researched. The theorem of existence and conditions of optimality are obtained.

Keywords:

Differential inclusion, ensemble of trajectories, optimal control problem, existence theorem, conditions of optimality.

1. Introduction.

Control and observation problems under conditions of uncertainty are models of real situations that take into account information constraints. They relate to special control problems for dynamic systems. In studies of such problems, much attention is paid to various properties of an ensemble of trajectories and a set of attainability, methods of forecasting and estimating the phase state, and minimax synthesis [1, 2].

Various classes of controlled differential inclusions and their discrete analogs can be used as a mathematical model of control systems under information constraints [3-10]. Differential inclusions find effective applications in the study of such important issues as the existence of optimal control, the dependence of optimal trajectories on the initial data and parameters, controllability of an ensemble of trajectories, necessary and sufficient conditions for optimality.

2. Statement of the problem. Consider a differential inclusion of the form

$$\dot{x} \in A(t)x + B(t,u,q),\ x(t_0) \in D, u \in V, q \in Q,\ t \in T_\infty = [t_0,+\infty), \qquad (1)$$

where x - n - state vector, u - m - control vector, q - k - vector (parameter), $A(t)$ – $n \times n$ - matrix, $B(t,u,q) \subset R^n$, $D \subset R^n$, $V \subset R^m$, $Q \subset R^k$.

A measurable m - vector function $u = u(t)$, $t \in T(u) = [t_0, t_1(u)]$, is called an admissible control for system (1) if $u(t) \in V$ almost everywhere on $T(u)$..

An admissible trajectory corresponding to an admissible control $u = u(t)$, $t \in T(u)$, and a parameter $q \in Q$ is called an absolutely continuous n-vector function $x(t) = x(t,u,q)$ that satisfies the differential inclusion (1) and the initial condition $T(u)$ almost everywhere on $x(t_0) \in D$.

We denote by $U(T_a)$ the set of all admissible controls defined on $T_a = [t_0, a]$. Let $H_{T_a}(u,q,D)$ be the set of all admissible trajectories corresponding to admissible control $u \in U(T_a)$, parameter $q \in Q$ and initial set D. Put $X_{T_a}(t,u,q,D) = \{\xi : \xi = x(t), x(\cdot) \in H_{T_a}(u,q,D)\}$. A multivalued mapping $X_{T_a}(t,u,q,D), t \in T_a$ is called an ensemble of trajectories of a differential inclusion (1) corresponding to a control $u \in U(T_a)$, a parameter $q \in Q$ and an initial set D. In [4,6], the continuity, closedness and convexity of multivalued mappings $(u,q) \to H_{T_a}(u,q,D)$ and $(t,u,q) \to X_{T_a}(t,u,q,D)$ were studied.

Definition. We will say that an admissible control $u \in U(T_a)$ and a parameter $q \in Q$ transfer the ensemble of trajectories of a differential inclusion (1) into a movable terminal set $Y(t)$ if there is a number $t(u,q) \subset T_a$ such that:

$$X_{T_a}(t(u,q),u,q,D) \subset Y(t(u,q)) \qquad (2)$$

Note that in definition $t(u,q)$ is the first moment of time when the ensemble of trajectories is transferred to the terminal set $Y(t)$, i.e. along with condition (2), it is assumed that the relation

$$X_{T_a}(t,u,q,D) \setminus Y(t) \neq \varnothing,\ t_0 \le t < t(u,q). \qquad (3)$$

Let U be the set of all admissible controls. We denote by $W(U,Q)$ the set of all admissible pairs $(u,q) \in U \times Q$ that translate the ensemble of trajectories of the differential inclusion (1) into the set $Y(t)$.

Relations (2) - (3) on the set $W(U,Q)$ define the functional $t(u,q)$. Consider the time optimal control problem: find a control $u^* \in U(T_a)$ and a parameter $q^* \in Q$ that minimize the functional $t(u,q)$, i.e.

$$t(u^*,q^*) = \inf\{t(u,q) : (u,q) \in W(U,Q)\}. \qquad (4)$$

By solving the time optimal control problem we mean the optimal pair (u^*,q^*) and the optimal time $t^* = t(u,^* q^*)$.

3. Main results. The control system (1) will be studied under the following assumptions: 1) the elements of the matrix $A(t)$ are summable on each segment $T_a \subset T_\infty$; 2) for any $t \in T_\infty$, $u \in V, q \in Q$ the set $B(t,u,q)$ is compact from R^n; 3) for any $T_a \subset T_\infty$ multivalued mapping $(t,u,q) \to B(t,u,q)$ is measurable in $t \in T_a$, continuous in $(u,q) \in V \times Q$ and there exists a function T_a summable on $\beta(t)$ such that $\sup\{\|\gamma\| : \gamma \in B(t,u,q)\} \le \beta(t)\}$, $(t,u,q) \in T \times V \times Q$; 4) for each $t \in T_\infty$, $\psi \in R^n$ the support function $C(B(t,u,q),\psi)$ ($C(P,\psi) = \sup_{p \in P}(P,\psi)$, $P \subset R^n$) is convex in $(u,q) \in V \times Q$; 5) for each $t \in T_\infty$ the set $Y(t)$ is convex and closed; 6) multivalued mapping $t \to Y(t)$ is continuous on any segment $T_a \subset T_\infty$; 7) $D \subset R^n$ are compact sets, and $V \subset R^m$, $Q \subset R^k$ are convex compact sets.

Under these conditions, the multivalued mapping $(t,u,q) \to X_{T_a}(t,u,q,D)$ is continuous on $T_a \times U(T_a) \times Q$, and the support function $C(X_{T_a}(t,u,q,D),\psi)$ is convex in $(u,q) \in U(T_a) \times Q$.

We put $W(T_a,U,Q) = W(U,Q) \cap (U(T_a) \times Q)$. It is clear that if $W(T_a,U,Q) \neq \varnothing$, then the optimal pair (u^*,q^*) is an element of the set $W(T_a,U,Q)$ and the optimal time t^* satisfies the equality

$$t^* = \inf\{t(u,q) : (u,q) \in W(T_a,U,Q)\}.$$

Theorem 1. If the set $W(T_a,U,Q)$ is nonempty, then a solution in the time optimal control problem (4) exists.

Now let us consider the functional

$$\mu_a(t,u,q) = \sup_{\|\psi\|=1}[C(X_{T_a}(t,u,q,D),\psi) - C(Y(t),\psi)],\ t \in T_a,\ u \in U(T_a).$$

The functional $\mu_a(t,u,q)$ is continuous on $T_a \times U(T_a) \times Q$ and convex in (u,q) on $U(T_a) \times Q$.

Theorem 2. In order for the control $u^* \in U(T_a)$, parameter $q^* \in Q$ and time moment $t^* \in T_a$ to be optimal in the problem (2), it is necessary and sufficient to satisfy the following conditions:

a) $\mu_a(t^*,u^*,q^*) = \min_{u \in U(T_a), q \in Q} \mu_a(t^*,u,q) = 0;$ (5)

b) t^* is the minimum root of the equation

$$\min_{u \in U(T_a), q \in Q} \mu_a(t,u,q) = 0, t \in T_a. \qquad (6)$$

4.Conclusion. Theorem 2 gives necessary and sufficient optimality conditions expressed in terms of a special functional $\mu_a(t,u,q)$. Under additional conditions with respect to the right-hand side $B(t,u,q)$ of the differential inclusion (1), from these optimality conditions (5) and (6) can be obtain the optimality conditions in the form of the supporting maximum principle. The results obtained can be used to study control systems under conditions of inaccurate information about the current state and external influences.

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